Back EMF Phase Relationships in Moving-Coil Loudspeakers (Part 3)

"Slip" Creates Inductive, Capacitive, or Resistive Back EMF

In the third article in this series, we discuss the concept of slip, in order to explain how and why it causes the impedance to be inductive, resistive, or capacitive, depending on frequency.

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Due to inertia, there is a delay in time between the motor force driving the cone (in phase with current) and the velocity of the coil as it moves through the magnetic field. This creates a phase delay between the input signal and the Back EMF, which is in phase with velocity ($V = Blv$). This phase delay is what causes Back EMF to simulate inductors, capacitors, and resistors. For the purposes at hand, we will refer to this delay in time between input current and cone position as "slip."

There is no established convention for the use of the term "slip." I have seen it used to describe the phase relationship between input voltage and Back EMF voltage.[1]

However, I find using input voltage as a reference in this context not to be useful, for two reasons. First, input current, as opposed to voltage, is precisely in phase with motor force, providing a link between the electrical and Newtonian systems. Input voltage is not, because voltage and current are shifted in phase with respect to one another whenever impedance is reactive. In other words, there is a phase shift between input voltage and current introduced by the reactive impedance before we can even approach the subject of the phase relationship between input signal and cone position. Second, Back EMF voltage is precisely in phase with voice-coil velocity, which is also a Newtonian quantity. These quantities have fixed differential and integral relationships to one another, which are lacking when input voltage is used as a reference. Input current, however, has a fixed integral relationship to cone position. For this reason, we will define "slip" to be the angle between input current and cone position, not the angle between input voltage and Back EMF voltage. This is why, in the second part of the article series "Thought Experiments," we used current as a reference, not voltage.

Calculation of Angle of Slip Between Input Current and Back EMF

We know the phase angle of the Back EMF from calculations described in the first article in this series (May 2018). The voltage of the Back EMF is reversed in polarity from its impedance, just as with the DC motor. The velocity of the cone is precisely in phase with Back EMF voltage. Velocity precedes position, because velocity is defined to be rate-of-change of position. Therefore, cone position lags behind velocity by 90° (see Figure 5 from the first part of the article series, May 2018).

So, we take the phase angle of the Back EMF impedance, subtract 180° to get the phase angle of the voltage, and then add 90° to represent the lag in time between velocity and position. The angle of Slip at a given frequency, then is calculated using the following equation:

$S = (\Theta - 90)$ using degrees,

where:

S = angle of slip
Θ = Back EMF Impedance Phase Angle

Figure 1 shows the angle of slip of the Fisher woofer (which is the example we have used throughout the article series) as a function of frequency. We see the angle of slip increasing as frequency is increased, with the most rapid increase in phase delay occurring near resonance. The angle of slip becomes more-or-less constant at higher and at lower frequencies, for reasons not described here. At very low frequencies, slip will approach zero, because there is very little delay between input and position. At DC, slip drops to zero.

Slip Causes Back EMF to Simulate LCR Components

A reactive load (capacitive or inductive) is characterized by its phase relationship between the input signal and the return of energy stored in the reactive component(s). In a loudspeaker driver, energy is stored in, and returned from, the moving mass of the cone assembly and the spring compliance. As this stored energy is returned, the voltages generated by the cone's motion (Back EMF) have phase relationships to input signal identical to those of the reactive components in the LCR model (see **Figure 2**).

To the source voltage, there is no difference between the LCR model and the moving-coil loudspeaker. The return of the energy to the system from the LCR model duplicates the return of energy from the mass and compliance of the loudspeaker, relative to input signal. It is the inertia of the cone, causing its motion to lag behind the input signal, which causes these phase relationships, resulting in the reactive impedance of the driver.

Back EMF and LCR Models Are Slightly Different

We can model the loudspeaker's impedance at any frequency using only two components: a resistor and a reactive component (inductor or capacitor). **Figure 3** shows the model at 23 Hz, below resonance. When we make a measurement of the complex impedance, we find a resistance in series with a reactance, as shown in Figure 3. (Above resonance, the reactive component would be a capacitor. The concept is the same.)

Figure 4 shows a more accurate representation of the moving-coil driver at this frequency. When Back EMF is involved, we can't perfectly model the loudspeaker using two components, because Back EMF itself simulates a resistor in series with another component (as shown in Figure 10 from the first part of the article series).

In a way, the two components making up Back

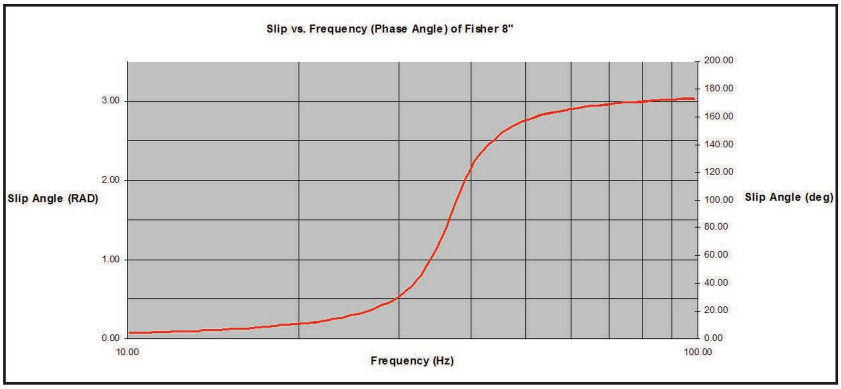

Figure 1: The phase angle of slip is shown between the input current and the cone position with respect to frequency.

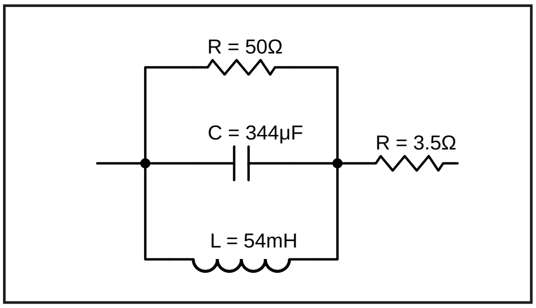

Figure 2: The LCR Model shows the equivalent real-world values of impedance for the Fisher woofer, which is used as an example throughout the article series.

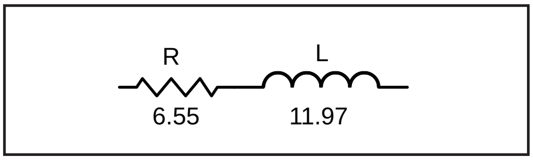

Figure 3: The resistor is shown in series with the inductor below resonance.

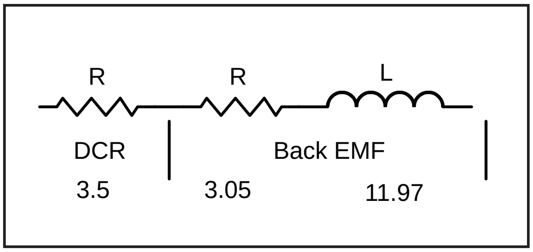

Figure 4: There is a distinction between voice-coil resistance and the resistive component of Back EMF.

EMF are in series and parallel at the same time. They share voltage, as if they were in parallel, but their impedances add as vectors, as if they were in series. Therefore, a loudspeaker's impedance can't be perfectly modeled using two components, as in Figure 3.

Inductors are the same way. They can't be perfectly modeled using a pure inductance and a pure resistance, but only using an infinite number of tiny components and integral calculus. Back EMF can be either capacitive or inductive, depending on frequency. Therefore, when comparing the behavior of Back EMF to that of the LCR model (shown in Figure 2), it is necessary to introduce a correction factor to compensate for the difference between the pure LCR models and the actual loudspeaker.

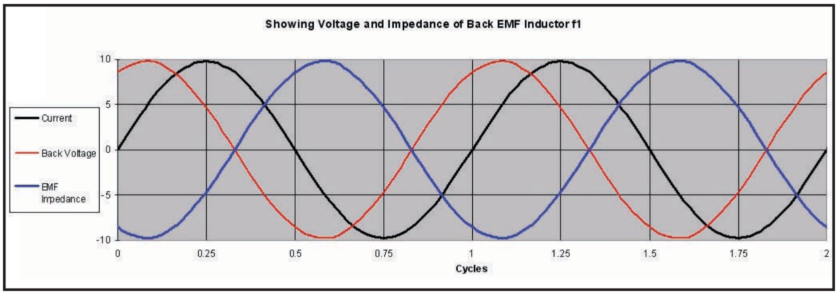

Figure 5: Here are the phase relationships of resistor/inductor model at f_1, 23 Hz.

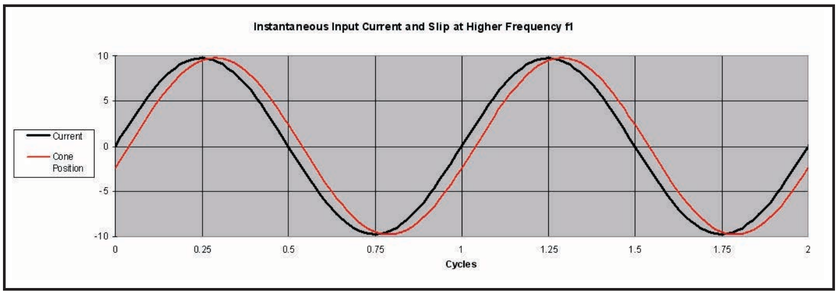

Figure 6: With a 14.4° Slip at test frequency, f_1 = 23 Hz.

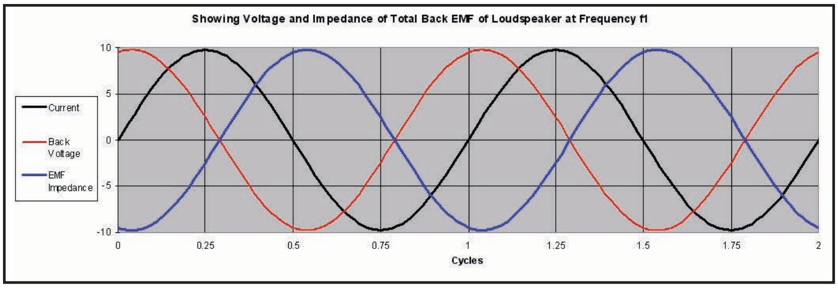

Figure 7: Here are the phase of Back EMF voltage and the loudspeaker's impedance at 23 Hz (f_1)

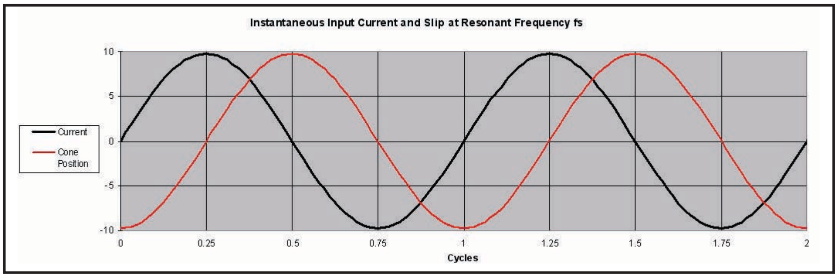

Figure 8: The slip at 90° (0.25 cycles) is at resonant frequency f_s, 36.93 Hz.

The correction factor is equal to the difference in phase angle between the pure reactive component and that of the Back EMF: 90 – 75.6 = 14.4° at f_1, (inductive), and -14.4 at f_2 (capacitive). These phase angles are sort of reciprocal to angle of slip ("sort of" because the relationships are not symmetrical below and above resonance—stay tuned).

Slip Angle of Back EMF Creates Inductive Reactance Below Resonance

We can graphically illustrate how, below resonance, the angle of slip creates an inductive reactance identical to that of the LCR model, or to that of the equivalent LR model. At f_1 (23 Hz), the impedance of our Fisher woofer can be modeled using a resistor with a resistance of 6.55 Ω in series with an inductor having a reactance of 11.97 Ω (see Figure 3).

We can illustrate the phase relationship of input current to the impedance of self-inductance (Back EMF) in this equivalent RL filter. We can see that the self-induced voltage precedes input current by 61.3°. The effective impedance of the self-induced EMF is 180° out-of-phase with voltage, as with any Back EMF. **Figure 5** shows the phase relationships occurring in the LR filter.

How Slip Creates Inductive Reactance

At the frequency f_1, 23 Hz, the angle of slip is 14.4°, as shown in **Figure 6**. Velocity precedes position by 90°; therefore, Back EMF precedes slip by the same amount. We know the angle of slip, so we can calculate the other relationships, which are shown in **Figure 7**.

Figure 7 is similar to Figure 5, but not identical. The curves representing the LR model show the Back EMF 61.3° behind current, and the curves illustrating the characteristics of the loudspeaker show Back EMF 75.6° behind current. The difference would be equal to the correction factor of 14.4° of the resistive and reactive components of Back EMF on the basis of their integrated nature, as previously explained.

Once the correction is made, the curve becomes identical to Figure 5, which shows the characteristics of the LR model. The phase position of the loudspeaker's Back EMF is the same as that of the self-inductance of the inductor in the LR model. Therefore, Back EMF is inductive, as is the driver's impedance.

Back EMF is Purely Resistive at Resonance

The speaker's impedance at resonance is purely

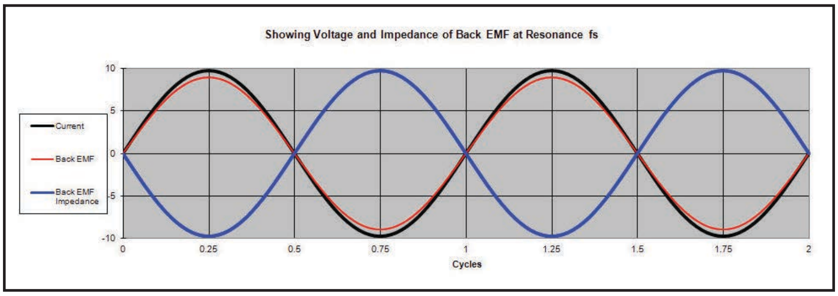

Figure 9: Back EMF impedance perfectly opposes the input signal at resonance.

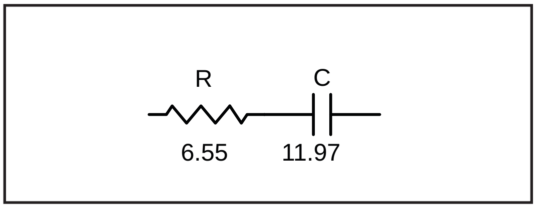

Figure 10: The angle of slip creates a capacitive resistance identical to the two-component RC model of impedance at f_2, 59.3 Hz.

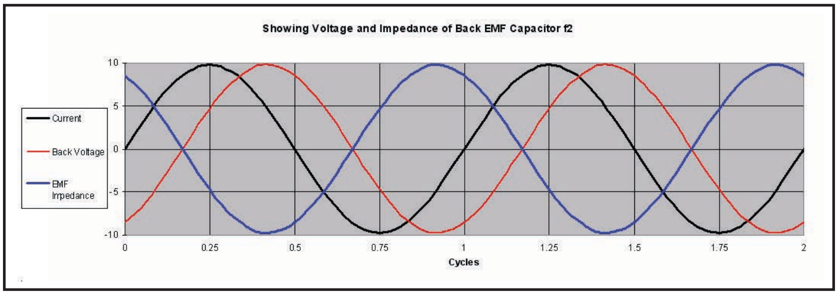

Figure 11: These are the phase relationships occurring in this equivalent RC filter at f_2.

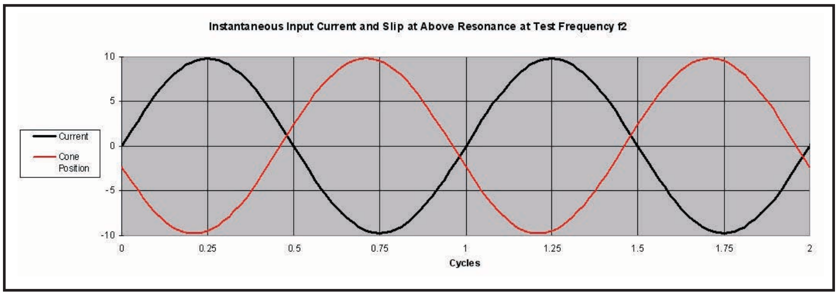

Figure 12: The 166° slip is shown at test frequency, f_2, 59.3 Hz.

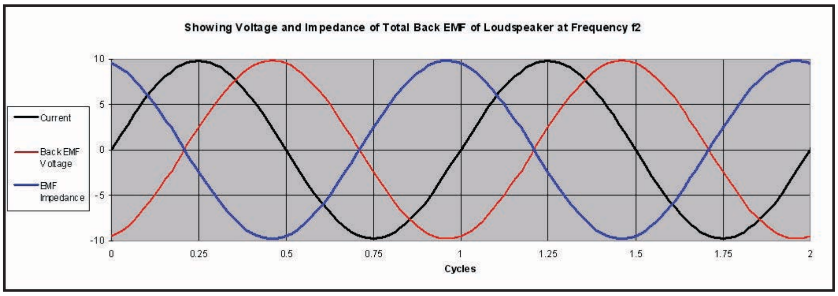

Figure 13: The phase of Back EMF voltage and the loudspeaker's impedance is shown at 59.3 Hz (f_2).

resistive, just as with the LCR model or the DC Rotary Motor (shown in Figure 2 in the first part of the article series). As always, Back EMF impedes the input voltage, but at resonance, the Back EMF is resistive, because of the angle of slip. The angle of slip has increased to 90°, as the mass of the cone causes it to lag farther behind input current (see Figure 1). The relationship between input current and cone position is shown in **Figure 8**.

Again, because velocity is in phase with Back EMF, and has a differential relationship to slip (position), it is possible to illustrate the relevant relationships graphically, as in **Figure 9**.

At resonance, the input current and voltage are in exact phase with one another, as with any purely resistive load. Cone position (slip), lags behind current by 90° (see Figure 1 and Figure 8). But because velocity (in phase with Back EMF) precedes position by 90°, the Back EMF is precisely in phase with input voltage and current. The resistance of the Back EMF presents a perfect resistive impedance to the input.

This is the exact relationship between the DC input current/voltage and the purely resistive Back EMF we see in the DC Motor, described in that section. There is no reactive component to Back EMF at resonance, and the resistor in the LCR model of Back EMF is equal to total driver impedance (resistance) minus the voice coil's DC resistance.

Slip Angle of Back EMF Creates Capacitive Reactance Above Resonance

We can illustrate how, above resonance, the angle of slip creates a capacitive reactance identical to that of that of the LCR model, or the two-component RC model (see **Figure 10**) at this frequency. The impedance characteristics are a mirror image of those at f_1, below resonance. The slip characteristics, however, are not symmetrical in an analogous way, but are part of the continuous increase of slip, with frequency, as shown in Figure 1.

At the frequency at f_2, 59.3 Hz, the impedance of our Fisher woofer can be modeled using a resistor with a resistance of 6.55 Ω in series with a capacitor having a reactance of 11.97 Ω.

The impeding voltage lags behind input current by 61.3°, as the capacitor charges. The impedance of this opposing voltage is 180° out-of-phase with voltage. **Figure 11** shows the phase relationships occurring in this equivalent RC filter.

References

[1] R. Carver, "How to Design a High-Pressure, High Back-EMF Subwoofer," 2002.

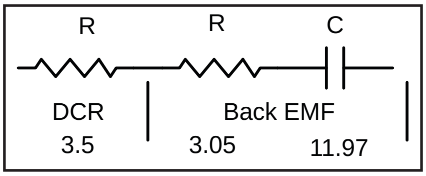

Figure 14: The dual parallel/series of the voltages on the components representing Back EMF changes the model slightly.

How Slip Creates Capacitive Reactance

At the frequency f_2, 59.3 Hz, the angle of slip is 166°. The position of the cone lags far behind input current, as shown in **Figure 12**. As always, velocity precedes position, so Back EMF precedes slip. We know the angle of slip, so we can calculate other relationships, as shown in **Figure 13**.

As was explained, the dual parallel/series nature of the voltages on the components representing Back EMF changes the model slightly, as shown in **Figure 14** (compare with Figure 4). As before, this necessitates the introduction of a correction factor, this time of -14.4°. This results in curves identical to those of two discrete RC components (see Figure 11), which need not be replicated here.

Once the correction is made, the curve becomes identical to Figure 11, which shows the characteristics of the RC model. The phase position of the loudspeaker's Back EMF is the same as that of the voltage present on the charging capacitor in the RC model. Therefore, Back EMF is capacitive, as is the driver's impedance of the driver, as a result.

Summary

The impedance of a moving-coil loudspeaker in the region of its resonance is essentially equal to the vector sum of its DC resistance and Back EMF. The position of the moving assembly relative to input signal ("slip") is what causes Back EMF to present an inductive, a resistive, or a capacitive component to the impedance.

To the driving voltage at the loudspeaker's terminals, the characteristics a loudspeaker's Back EMF are indistinguishable from a model of the driver made of discrete LCR components, the values of which can be specified.

The inductive reactance associated with the increasing impedance of a loudspeaker below its resonance is not as a result of self-inductance, as in the case of an actual inductor, but rather of the phase relationship between the input voltage and Back EMF. The driver's impedance is purely resistive at resonance because the Back EMF itself has a phase angle of zero. Back EMF and voice-coil resistances can be added as scalars at resonance.

The capacitive reactance associated with the decreasing impedance of a loudspeaker below its resonance is not as a result of a self-inductance, as in the case of an actual inductor, but rather of the phase relationship between the input voltage and Back EMF.

About the Author

Andy Lewis lives in Englewood, CO, plays drums, and produces "Acme Sound Low-B" low-coloration loudspeakers for bass players. He is also involved in Special Olympics and other disability activities with his son, Collin.