Frequency Tunable Analog Filters: Application to a Linkwitz-Riley Crossover

Numerical filters provide the ability to tune the cut-off frequency, but the anti-aliasing filter must be frequency tunable as well. Though switched capacitors filters allow that, one may prefer a pure analog filter due to noise, distortion or subjective issues. This section presents a method to easily and systematically perform cut-off frequency tuning on any kind of analog filter. A popular LR4 crossover filter using this method will be described.

The method presented here consists of a double approximation. The first approximation is the commonly applied approximation of known analog filters with a numerical system: the following approximation of the jω operator is applied to synthesize the numerical filter:

$$j\omega \approx \frac{2}{T_e} \times \frac{1-z^{-1}}{1+z^{-1}} \qquad [1]$$

This approximation turns out to be satisfying if Fc<<Fe.

The second approximation consists of approximating the numerical filter with an analog filter. One may approximate pure delays with analog first-order all-pass structures:

$$z^{-1} = e^{-j\omega T_e} \approx \frac{1 - j\omega \times {T_e}/{2}}{1 + j\omega \times {T_e}/{2}} \qquad [2]$$

Combining the approximations from 1 and 2 results in an exact approximation of jω:

$$j\omega \approx \frac{2}{T_e} \times \frac{1-z^{-1}}{1+z^{-1}} \approx \frac{2}{T_e} \times \frac{1 - \dfrac{1 - j\omega \times {T_e}/{2}}{1 + j\omega \times {T_e}/{2}}}{1 + \dfrac{1 - j\omega \times {T_e}/{2}}{1 + j\omega \times {T_e}/{2}}} =$$

$$\frac{2}{T_e} \times \frac{1 + j\omega \times {T_e}/{2} - 1 + j\omega \times {T_e}/{2}}{1 + j\omega \times {T_e}/{2} + 1 - j\omega \times {T_e}/{2}} = \frac{2}{T_e} \times j\omega \times {T_e}/{2} = j\omega \qquad [3]$$

Approximation [1] is a mathematical approximation.

Approximation [2] may be achieved in practice with a simple circuit (**Figure A**):

$$\underline{T}(j\omega) = \frac{1 - j\omega \times {T_e}/{2}}{1 + j\omega \times {T_e}/{2}} = e^{-2 \times arctg\left(\frac{\omega T_e}{2}\right)}$$

So, we have $\frac{T_e}{2} = RC$ and R, C should be the same for all occurrence of this circuit.

If you apply this method of complementary approximations, you can get frequency cut-off tuning by changing the identical resistors R of all the necessary first order all-pass structures, and this can be achieved with numerically controlled analog potentiometers or switched capacitors.

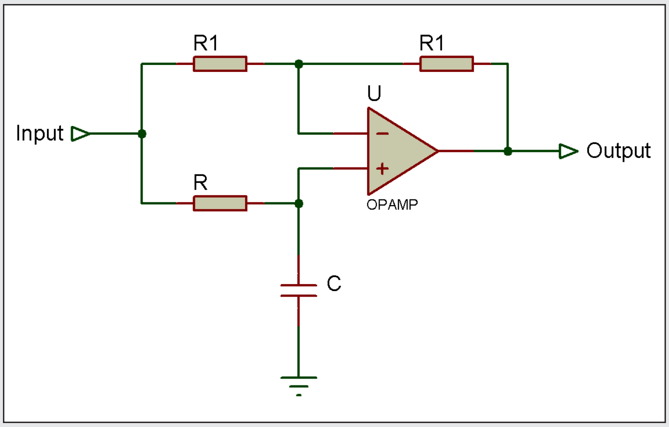

Figure A: A pure delay first-order approximate.

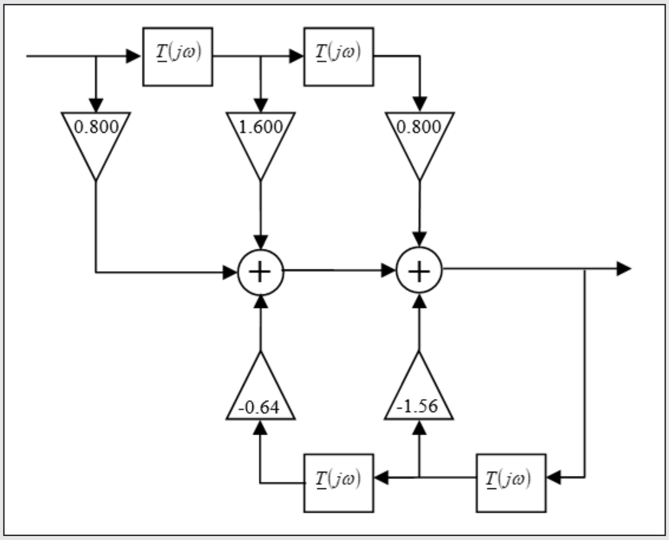

Figure B: A structure for a second-order Butterworth low-pass filter.

Let’s take the example of the second-order Butterworth low-pass:

$$\underline{H}(j\omega)=\frac{1}{1+2m\frac{j\omega}{\omega_0}+\left(\frac{j\omega}{\omega_0}\right)^2}\approx\frac{1}{1+2m\times\frac{2}{\omega_0T_e}\times\frac{1-z^{-1}}{1+z^{-1}}+\left(\frac{2}{\omega_0T_e}\times\frac{1-z^{-1}}{1+z^{-1}}\right)^2}$$

with:

$$2m=\sqrt{2} \tag{4}$$

Choose:

$$\frac{T_e}{2}=T_0=RC$$

. This is not a good choice for a numerical filter, but it is suitable for our method, where phase rotation of the pseudo-delays is limited to π.

The numerical approximation leads to:

$$\underline{H}(z)\approx\frac{\left(1+z^{-1}\right)^2}{\left(1+z^{-1}\right)^2+\frac{m}{\pi}\times\left(1-z^{-1}\right)\times\left(1+z^{-1}\right)+\left(\frac{1}{2\pi}\right)^2\times\left(1-z^{-1}\right)^2}$$

$$\underline{H}(z)\approx\frac{1+2\times z^{-1}+z^{-2}}{\left(1+\frac{m}{\pi}+\frac{1}{4\pi^2}\right)+\left(2-\frac{1}{2\pi^2}\right)\times z^{-1}+\left(1-\frac{m}{\pi}+\frac{1}{2\pi^2}\right)\times z^{-2}}$$

Then:

$$\left(1+\frac{m}{\pi}+\frac{1}{4\pi^2}\right)\times S_n+\left(2-\frac{1}{2\pi^2}\right)\times S_{n-1}+\left(1-\frac{m}{\pi}+\frac{1}{4\pi^2}\right)\times S_{n-2}=E_n+2\times E_{n-1}+E_{n-2}$$

$$S_n=-b_1\times S_{n-1}-b_2\times S_{n-2}+a_0\times E_n+a_1\times E_{n-1}+a_2\times E_{n-2}$$

$$\begin{cases} a_0=\dfrac{4\pi^2}{\left(4\pi^2+4m\pi+1\right)}\approx 0.800 \\ a_1=\dfrac{8\pi^2}{\left(4\pi^2+4m\pi+1\right)}\approx 1.600 \\ a_2=\dfrac{4\pi^2}{\left(4\pi^2+4m\pi+1\right)}\approx 0.800 \\ b_1=\dfrac{\left(8\pi^2-2\right)}{\left(4\pi^2+4m\pi+1\right)}\approx 1.560 \\ b_2=\dfrac{\left(4\pi^2-4m\pi+1\right)}{\left(4\pi^2+4m\pi+1\right)}\approx 0.640 \end{cases}$$

The corresponding filter is shown in **Figure B**.

Other implementation methods may be used, the important issue being how the filter will behave with respect to components precision.

If you now double the structure of Figure B you get a cut-off frequency-tunable fourth-order Linkwitz-Riley low-pass!