Back EMF Phase Relationships in Moving-Coil Loudspeakers (Part 4)

Back EMF Reactive Impedances and Energy Losses

In this article, Andy Lewis expands upon the Back EMF concepts hinted at in the previous three articles in this series. The theories can be applied to virtually all areas of loudspeaker design, starting with two filters: Simultaneous Electrical and Newtonian Systems.

By
Andy Lewis
(Acme Sound, LLC)

In the first installment of this series, which ran in the May 2018 issue of *audioXpress*, we identified and quantified Back EMF at and around the resonant frequency of a moving-coil loudspeaker, and modeled it using a simple LCR filter, as shown in **Figure 1**.

In Figure 1, the 3.5 Ω resistor represents the voice coil's DC resistance. The RCL parallel filter models the Back EMF, as previously described. A moving-coil loudspeaker, however, is not composed of one resonant filter, but of two. It is not a coincidence that these two resonant filters share the same frequency of resonance. It was observed in the first article in this series (*audioXpress*, May 2018) that:

$$LC = MC_{ms} \qquad (1)$$

where:

L = inductor in LCR model
C = capacitor in LCR model
M = mass of loudspeaker moving assembly
C_{ms} = loudspeaker's compliance

We will examine this convergence.

What are Electrical and Newtonian Filters?

Because the previous articles in this series were about Back EMF specifically, they overlooked the primary resonant filter, which is made up of the loudspeaker's moving mass and compliance. We will call this resonant filter the Newtonian Filter. These physical characteristics of the driver determine its free-air resonance, according to this common equation:

$$f_s = \frac{1}{2\pi\sqrt{(MC_{ms})}} \qquad (2)$$

where:

f_s = frequency of resonance
M = moving mass
C_{ms} = compliance

Because the loudspeaker's resonant frequency is determined by these Newtonian properties of mass and compliance, the resonant frequency is, as well. The filter created by Back EMF is modeled using a resistor, a capacitor, and an inductor (see Figure 1). We will refer to this LCR filter as the Electrical Filter (Back EMF). The resonant frequency of this filter is necessarily the same as, and determined by, the characteristics of this Newtonian filter, as in Equation 2. For the purposes at hand, the terms "Electrical Filter" and "Back EMF" can be considered interchangeable. The equation relating capacitance, inductance, and resonant frequency of an electrical filter is:

$$f_s = \frac{1}{2\pi\sqrt{(LC)}} \qquad (3)$$

where:

f_s = frequency of resonance
L = inductance
C = capacitance

The two equations, 2 and 3, are obviously analogous. This is because the reactive Newtonian quantities are an analog to the electrical reactive components making up the LCR filter. We will examine the intimate relationship between these two filters.

An electrical LCR filter has both a capacitance and an inductance. A loudspeaker's physical structure has both a physically inductive moving mass and physically capacitive spring compliance. We call the moving mass inductive because, like an inductor, it has a high reactance at high frequencies, in that it's difficult to accelerate at high frequencies. We call the loudspeaker's spring capacitive because, like a capacitor, it has a high reactance at low frequencies. Both of these physical components store energy, and return it to the system, just as do the electrically-reactive components making up the Electrical Filter.

The Electrical Filter

We have seen that the impedance of a loudspeaker can be modeled using passive components. From the point of view of an AC voltage, the speaker exhibits inductive and capacitive reactances and resistance, just as the LCR model does, and the impedance curves are nearly identical (see **Figure 2**)

Capacitors and inductors are devices which store energy and release it back to the system. This energy is stored in the form of concentrated electric charge in capacitors, and in the form of external magnetic fields in and around inductors. As the frequency applied to a loudspeaker driver is increased, the impedance of the unit increases below resonance, just at the impedance of the LCR model does. (For more detailed explanations see the previous articles in this multi-article series, which ran in *audioXpress* May, June, and July 2018.) This rising impedance is, by definition, inductive. Similarly, above resonance, the impedance is decreasing, which is characteristic of a capacitance.

As we have seen, Back EMF behaves as if it were an electrical filter, even in the absence of these reactive and resistive passive components. In the LCR model, energy is dissipated as heat by the series resistor representing the DC resistance of the voice coil and by the resistive component of the LCR model. At any frequency, the energy

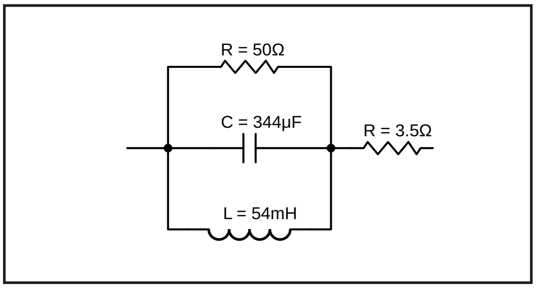


Figure 1: Specific values of the LCR model of a Fisher woofer

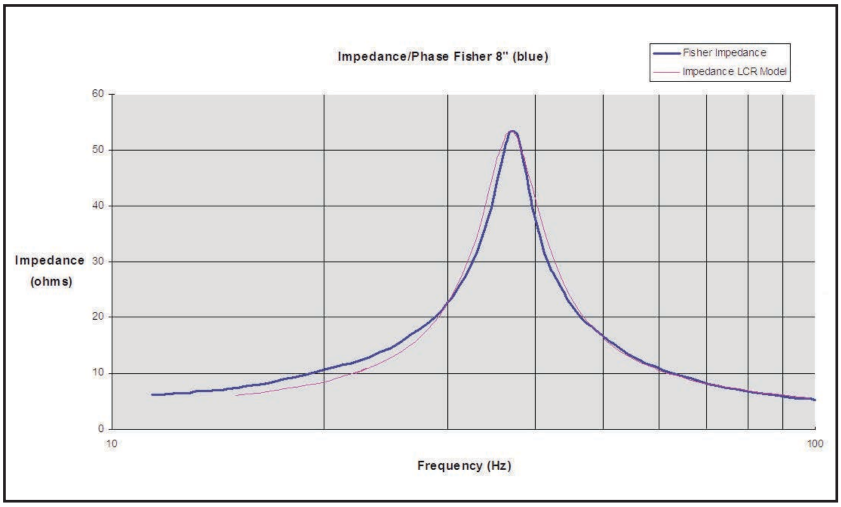


Figure 2: This graph shows the impedances of the loudspeaker and of the LCR model.

lost to heat is equal to the RMS current squared times the total DC resistance:

$E = I^2R$ (4), also used below

where:

E = energy lost to heat
I = RMS current
R = DC Resistance

The Newtonian Filter

In the LCR model, energy storage and retrieval are to and from the electrostatic and the magnetic fields of capacitors and inductors. The components of the physical loudspeaker assembly also store and release energy, but through a different mechanism—energy is stored in and released from a different type of reactive filter.

This filter and its storage of energy involves quantities we associate with Isaac Newton and his Laws of Motion: mass, velocity, and spring characteristics. In the moving-coil loudspeaker, energy is stored in, and released from, the moving mass and the spring compliance. As a speaker cone moves back and forth, kinetic energy is stored in, and returned from, the moving mass. Similarly, potential energy is stored in, and returned from, the surround and the spider, as a speaker's moving assembly oscillates.

As this activity is occurring, some energy is lost to frictional heat. Frictional losses can be seen as resistive components, because they dissipate heat, just as a resistor does. More on this later. The moving mass of a loudspeaker's cone and the spring force of the compliance result in a physical resonance, just as a capacitor and inductor form an electrical one.

Just as the electrical filter is made up of a reactive capacitor and a reactive inductor, the Newtonian filter is made up of a reactive mass, and a reactive compliance (with frictional resistance). Just as an electrical inductance impedes high frequencies in the electrical filter, the inductive nature of the loudspeaker's moving mass impedes the cone's high-frequency acceleration. This can be seen as a physical inductance.

Similarly, the loudspeaker's compliance impedes the cone's low-frequency acceleration, just as a capacitor impedes low frequencies in the electrical filter. This can be seen as a physical capacitance. As energy is stored in, and returned from, the mass and the compliance, it results in impedance characteristic essentially identical to that of the electrical filter made up of a capacitor, an inductor, and a resistor. This Newtonian filter has a resonant frequency. In the world of loudspeakers, this is referred to as f_s. The equation relating mass, compliance, and resonant frequency of this physical filter is shown in Equation 2.

Intersection of Two Filters

Because Electrical and Newtonian filters share a common resonant frequency, they have a mathematical similarity:

$$f_s = \frac{1}{2\pi\sqrt{(LC)}} = \frac{1}{2\pi\sqrt{(MC_{ms})}} \quad (5)$$

and algebraically that:

$$\frac{1}{\sqrt{(LC)}} = \frac{1}{\sqrt{(MC_{ms})}} \quad (6)$$

And ultimately:

$$LC = MC_{ms} \quad (7)$$

This quantity is expressed in "seconds squared." The resonant frequencies of the two filters coincide. But because f_s is determined by the physical quantities of mass and compliance, f_s is not dependent on the electrical filter, but only the Newtonian one. Obviously, then, as you add mass to the cone, the effective capacitance and the inductance of the electrical filter change to equal the lower resonant frequency of the altered Newtonian filter.

Similarly, when you install the driver in a sealed enclosure, the electrical filter would have to change effective component values to model the decreased compliance, and the increase in Newtonian resonant frequency, in turn.

Observation on Reactive Parallel Electrical Filters as Pertains to Back EMF

We have previously shown that the Back EMF of the Fisher woofer can be modeled by using a parallel filter made up of a 50 Ω resistor, a 344 µf capacitor, and a 54 mH inductor. The DC resistance of the voice coil is 3.5 Ω.

At f_1, 23 Hz, the inductive reactance of the parallel LCR filter in is 11.97 Ω, and the effective

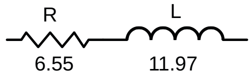


Figure 3: This is the RL model of the Fisher woofer at 23 Hz.

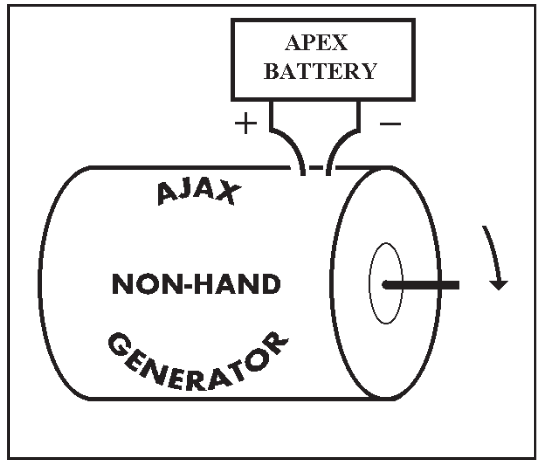


Figure 4: A "Generator" driven by an external voltage is a motor.

DC resistance is 6.55 Ω. Because, at this frequency, the impedance is inductive, it can be modeled by a simple resistor and inductor, as in **Figure 3**.

Yes, you read correctly. At 23 Hz, the impedance of the two components in Figure 3 is the same as that of the more complex filter in Figure 1, and as that of the Fisher woofer itself! At this frequency, this inductive reactance represents an inductance of an inductor of 82.8 mH. What is interesting is that while the actual inductor in the LCR filter is 54 mH (see Figure 1), the apparent inductance at that frequency is much greater.

This is due to the bizarre nature of parallel reactive filters, in which currents add as vectors. Therefore, because the currents in the capacitor and in the inductor are traveling in opposite directions at any instant, the phase angle between their impedances is 180°. The currents are exactly out of phase with one another. Therefore, below resonance, the capacitor in the LCR model has an effective negative-capacitance, which is equivalent to a positive inductance, and therefore adds to the net inductance of the filter. In other words, the opposite current in the capacitor offsets the current in the inductor, resulting in lower net current. Less current equals higher impedance (Ohm's law). Therefore, yes, it is possible for the filter's effective inductance to be greater than the actual inductance of the inductor, even though it's completely counter-intuitive. Because of this, the effective inductance of the LCR filter below resonance will always be greater that the inductance of the physical inductor itself.

Similarly, because of the "mirror image" nature of capacitors vs. inductors, the capacitance of the

LCR filter above resonance will always be less than that of the actual capacitor in the LCR model. In this case, at f_2 = 59.3 Hz, the capacitance due to the 11.97 Ω capacitive reactance is 224 µF, for the same reason: Opposite parallel currents result in less net current, which translates to higher impedance.

Energy Lost to Back EMF: Another Look at the DC Motor

In order to examine how energy is lost to Back EMF in a loudspeaker, it is helpful to return to the model of the DC rotary motor, that we used as in the first article of this multi-part series (*audioXpress*, May 2018). We will use the same motor we used as our previous example and will drive this motor with no external load. The only physical resistance to its rotation will be in the friction of its bearings, which warm slightly as heat is generated by this friction.

The total resistance of the DC motor is equal to the internal resistance of the motor plus the effective resistance of the Back EMF generated as the motor turns (see article 1 of series). Back EMF manifests itself as, and can be expressed and measured as, a resistance. When this resistance is added to the motor's inherent resistance, the sum equals the total resistance of the working motor, as shown in **Figure 5**.

$$R_1 + R_2 = R_t \tag{8}$$

Add Friction or Load to the DC Motor: Increased Load Results in Decreased Back EMF

When there is no physical load on the motor, it turns very quickly for a given DC voltage. Because voltage is proportional to velocity, Back EMF is at its maximum, as is total resistance, in turn. Because of this high resistance, current flow is very low. Consequently, very little energy is dissipated as heat by the wire windings, or the bearings. In the case of a low-friction motor with no load, very little heat is generated, as very little energy is lost.

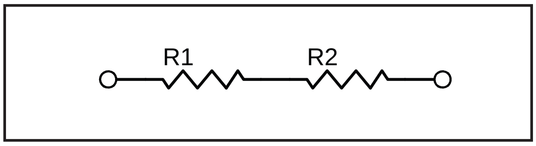


Figure 5: The DC motor is represented as two resistors.

With no load, the Back EMF has a greater resistance than the windings themselves, and more energy is lost to the Back EMF than to the heat dissipated by the windings (I^2R). The heat dissipated by the Back EMF component of the motor's resistance is in the friction of the bearings. The DC Motor, when not presented with an external load, is a device that dissipates very little heat because of its maximized resistance and low current flow.

If the motor is called upon to drive an external load, such as a fan, the external load will make the motor harder to turn. Internal friction has the same effect. Any frictional loss will result in heat, either within the motor itself, or externally, such as in the air being stirred up by the fan (this observation is relevant to loudspeakers). When the friction increases, the Newtonian motor force driving the coils is opposed by this friction. This decreases the net force on the rotating armature, for a given current flow, decreasing its velocity. Because Back EMF is proportional to velocity, less velocity equals less Back EMF, resulting in less resistance at the terminals. Therefore, Back EMF becomes a smaller component of total electrical resistance.

As frictional force increases to infinite (enough, in other words, to cause all activity to cease), Back EMF becomes zero, and the total resistance becomes equal to the resistance of the windings themselves. The result is maximum current flow, heat dissipation in turn, and risk of fire. That's why loading a motor down with a greater load than it can handle can burn it up; because of the loss of Back EMF.

Under normal circumstances, with the DC motor driving a reasonable load, the heat energy dissipated is equal to that lost to current through the resistance of the coils, the heat dissipated in the bearings by the resistance of the Back EMF, plus the heat dissipated by the load. The all-encompassing term "load," would include internal friction, and whatever external entity is being driven by the motor (e.g., a wheel, a propeller, or a lawn mower). The motor does Newtonian work on these loads, and heat is dissipated externally as a consequence, to various losses.

To summarize: For the DC Motor, when the load is minimized, Back EMF is at its maximum, and DC current is at its minimum. As frictional losses increase (including externally) they are manifested in a decreased Back EMF component

About the Author
Andy Lewis lives in Englewood, CO, plays drums, and produces "Acme Sound Low-B" low-coloration loudspeakers for bass players. He is also involved in Special Olympics and other disability activities with his son, Collin.

of the motor's total resistance. Current is increased proportionally, as described by Ohm's Law:

$I = E/R$ (9)

But while current is inversely proportional to total resistance, energy dissipated (power) is proportional to the square of the current.

$E = I^2R$

where:

E = energy
I = current
R = resistance

Ironically then, energy lost to friction, reflected in decreased Back EMF, actually increases total energy lost to that Back EMF, because lower resistance increases current. To summarize, all energy losses not associated with the I^2R energy lost to coil resistance is manifested in the resistive component of the Back EMF ($R_1 + R_2 = R_t$).

The resistance of the Back EMF decreases with increased load or internal friction, even as total energy lost to this friction increases. All of these things are true of the moving-coil loudspeaker, which we referred to as the "AC Linear Motor," in previous articles in this series.

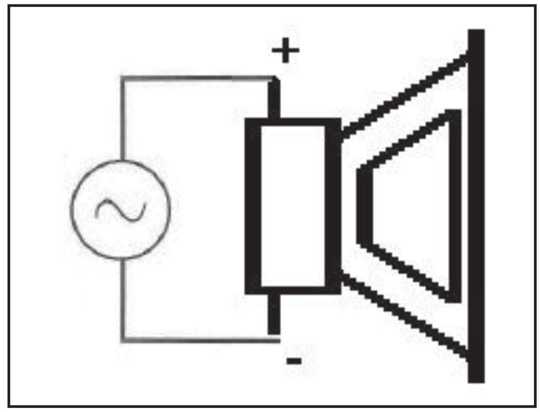


Figure 6: In the moving-coil loudspeaker the AC linear generator is driven by external voltage.

How Does This Translate to the Loudspeaker?

Figure 6 shows the moving-coil loudspeaker. To a voltage source, just as with the DC Motor, all energy losses are resistive and seen as the resistive

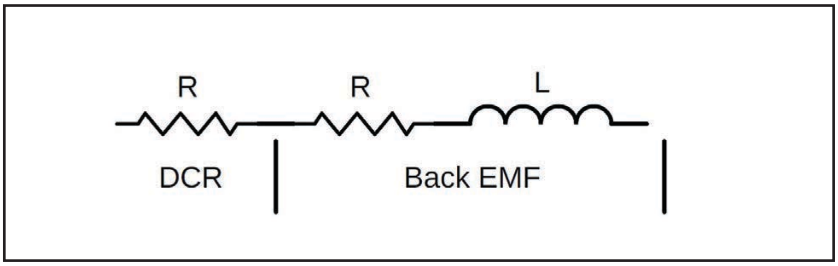


Figure 7: R_{DCR} and $R_{Back\ EMF}$ are shown in series with a reactive component of Back EMF.

component of the total impedance. This applies to the DC Motor, the moving-coil loudspeaker, and the LCR model of the loudspeaker's impedance. All energy (heat) dissipated by these systems is reflected in the resistive component(s) of their impedances.

The total resistance (not impedance) of the moving-coil loudspeaker is the sum of its voice-coil resistance and the resistive component of the Back EMF. To an amplifier, there is no difference between the resistive component of Back EMF and an actual resistor. Therefore, the energy not lost to voice coil DC resistance is necessarily dissipated by the resistive component of Back EMF, just as with the DC Motor, or any two resistances in series, as shown in **Figure 7**. Therefore:

$$E_t = E_{DCR} + E_{Back\ EMF} \quad (10)$$

and

$$E_{Back\ EMF} = E_t - E_{DCR} \quad (11)$$

where:

E_t = total energy dissipated

E_{DCR} = energy lost to voice coil resistance

$E_{Back\ EMF}$ = energy lost to the resistive component of Back EMF

References

A. Lewis, "Back EMF Phase Relationships in Moving Coil Loudspeakers (Part 3): "Slip" Creates Inductive, Capacitive, or Resistive Back EMF," *audioXpress*, July 2018.

———, "Back EMF Phase Relationships in Moving Coil Loudspeakers (Part 2): Thought Experiments, *audioXpress*, June 2018.

———, "Back EMF Phase Relationships in Moving Coil Loudspeakers (Part 1): Identifying, Quantifying, and Modeling Back EMF, *audioXpress*, May 2018.

Where Does Energy Lost to Back EMF Go?

Energy can neither be created nor destroyed, but can be converted to another form. In electrical motors it is converted to heat. In the case of the moving-coil loudspeaker, some of this heat is dissipated in the room, as your favorite music excites the air with sound waves. In the loudspeaker, as with the DC Motor, energy is dissipated as heat through frictional losses. Mechanical losses involve friction in various forms. Energy is lost in the surround, for example, and even to "wind" in the vicinity of the driver.

Some of these energy losses are audible, such as air being pushed through the gap, or through a vent in the dustcap. Air leaks in the enclosure behave similarly, and most are audible. If damping materials are used in an enclosure, energy is lost to this absorptive material, and is dissipated as heat. The reproduction of music warms the air in the room ever so slightly. All of these losses are manifested in the resistive component of the impedance.

It is possible to analyze the activity through a current cycle at any frequency to see when, and to what extent, the Newtonian force from the voice coil is opposed by the motion of the cone. At these moments in the current cycle, Newtonian work is done, the cone is slowed, and energy is dissipated. In the specific case of energy lost to this Newtonian work, the heat is dissipated in the voice coil by the resistive component of Back EMF. Just as an increased external load causes the wire windings of the DC Motor to get hotter, the work done during various parts of the current cycle cause the current through a loudspeaker's voice-coil to increase, and heat is dissipated, in turn.

What the DC Motor and the AC Loudspeaker have in common is that these energy losses all result in an increased physical load to the AC motor. This load decreases velocity, for a given input voltage, resulting in decreased Back EMF. The impedance seen by the voltage-source is decreased as Newtonian work is increased. Just as with the DC Motor, the impedance drops, resulting in increased current, and more heat dissipated into the environment.

Differences Between DC Motor and Loudspeaker

There is no energy storage in the bearings of a rotary motor, and no energy returned to the system. The impedance of the DC Motor is purely resistive, and much less complicated, as described in previous articles in this series. As an alternating device, the loudspeaker driver is

The “Must Have” reference for loudspeaker engineering professionals.

Home, Car, or Home Theater!

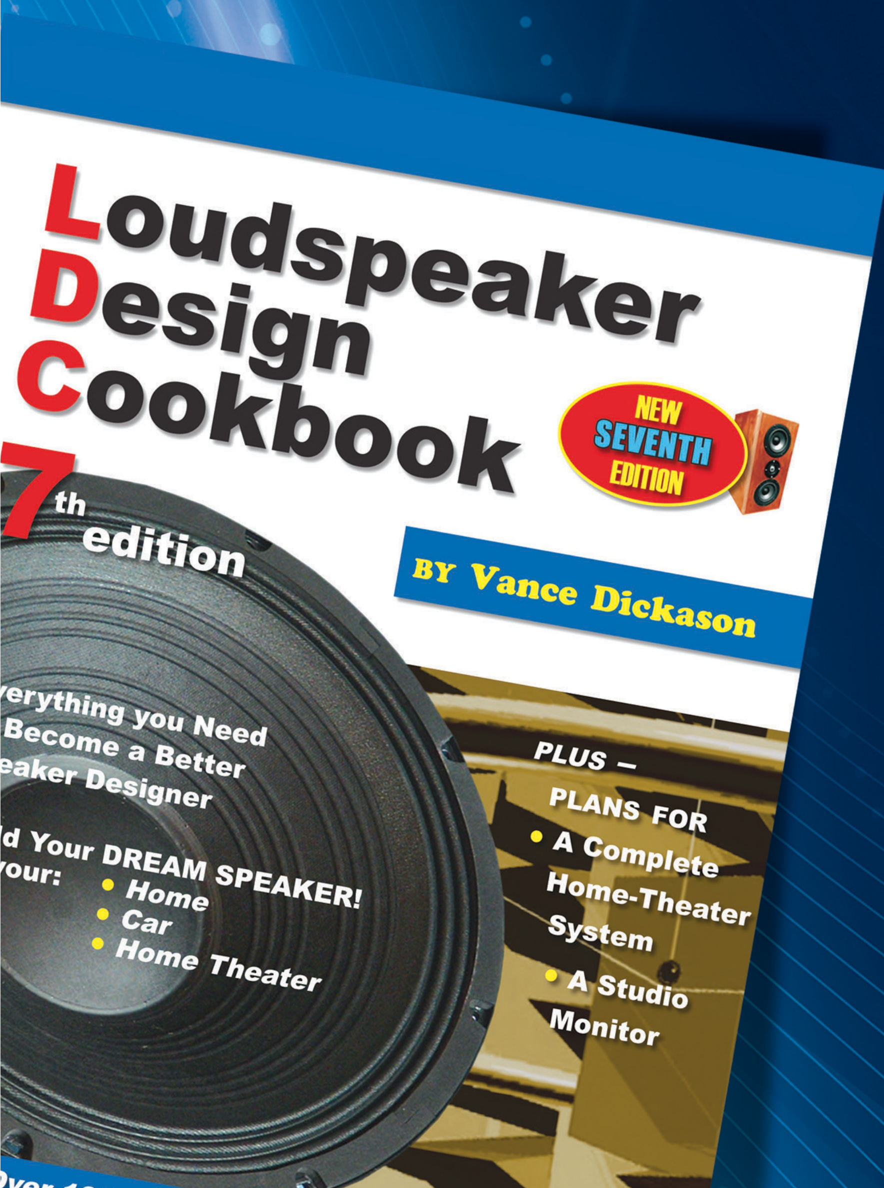
Back and better than ever, this 7th edition provides everything you need to become a better speaker designer. If you still have a 3rd, 4th, 5th or even the 6th edition of the *Loudspeaker Design Cookbook*, you are missing out on a tremendous amount of new and important information!

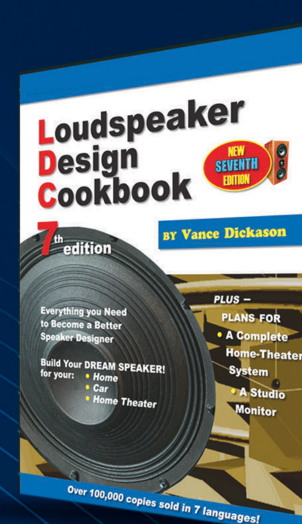
Now including: Klippel analysis of drivers, a chapter on loudspeaker voicing, advice on testing and crossover changes, and so much more! Ships complete with bonus CD containing over 100 additional figures and a full set of loudspeaker design tools.

A $99 value!

Yours today for just $39.95.

Shop for this book, and many other audio products, at **www.cc-webshop.com.**





	f_1	f_s	f_2
Frequency	23 Hz	36.93 Hz	59.3 Hz
Z_t	13.64 Ω	53.5 Ω	13.64 Ω
Z_t Phase (RAD)	1.07	0	-1.07
Z_t Complex Impedance (ohms)	6.55 + j11.97	53.5 +j0	6.55 - j11.97
EMF Complex Impedance (ohms)	3.05 + j11.97	53.50 + j0	3.05 - j11.97
EMF Magnitude	12.35 Ω	50 Ω	12.35 Ω
EMF Phase Angle (RAD)	1.32	0	-1.32
R_t	6.55 Ω	53.5 Ω	6.55 Ω
$R_{Back\ EMF}$	3.05 Ω	50 Ω	3.05 Ω
Voltage Applied	2.83 V	2.83 V	2.83 V
Current ($I = E/Z_t$)	0.207 A	0.053 A	0.207 A
Total Heat Dissipated I^2R	**0.282 W**	**0.15 W**	**0.282 W**
Dissipated by DCR	**0.151 W**	**0.01 W**	**0.151 W**
Dissipated by EMF	**0.131 W**	**0.14 W**	**0.131 W**

Table 1: These methods are used to isolate the resistive component of Back EMF at three frequencies and calculate the heat dissipated by Back EMF at these frequencies.

different from the DC Motor, in that the mass and compliance of the moving-coil loudspeaker have the ability to store and return energy. This results in the reactive component of Back EMF, and the high impedance peak we observe at resonance.

Electrical energy loss occurs when input current does Newtonian work on the cone, accelerating it in one direction or the other, or on the suspension, which alternately works with, or against the motor force. As with the DC Motor, an increased physical load will increase its I^2R loss as (electrical) resistance decreases and current increases, as was explained.

As with the DC Motor, there is work being done as the motor force is physically resisted variously by the inertia of the mass, the spring behavior of the suspension, and the various frictional losses. One of these frictional losses, thankfully, is the heat dissipated into the room as sound is produced by the speaker. (In an unpublished appendix, I have followed the moving assembly through its current cycle in each of the three modes of Back EMF generation, to illustrate how and when this Newtonian work is being done. Because there are three modes of Back EMF generation, as described in the first article of this series, three analyses are required to explore how and when this work is done by the loudspeaker in each of the three modes.)

Calculating Energy Lost to Back EMF

We have identified the loss of energy to the resistive component of Back EMF as a result of both frictional losses and Newtonian work. It can be interesting to see how much energy is lost to Back EMF at various frequencies. I have calculated these losses as they occur in our Fisher woofer. (Note that this woofer is in free-air. If the driver were actually mounted in an enclosure, there would be myriad other losses introduced. As was stated, each of these losses would ultimately be evident in the restive component of the impedance.)

Just as with any current flowing through a resistance, the energy dissipated is equal to the resistance multiplied by the square of the current (reactive components return energy, as opposed to dissipating it):

$$E = I^2R_{EMF} \quad (12)$$

where:

E = dissipated heat in watts
I = current in amperes
R_{EMF} = resistive component of Back EMF in ohms

The chart shown in **Table 1** uses the methods outlined in the previous articles in this series to isolate the resistive component of Back EMF at three frequencies, and calculates the heat dissipated by Back EMF at these frequencies. We arbitrarily use an input voltage of 2.83 V.

Interestingly, although the impedance of the speaker varies widely (from 13.64 Ω to 53.5 Ω), the energy dissipated by Back EMF is minimally changed! The difference between 0.140 W and 0.131 W is about 9.4%, or 0.277 dB, if you prefer. As the resistance of the Back EMF increases, the attendant decrease in current effectively offsets it, such that the power dissipated by the resistive component of Back EMF remains nearly constant.

In Closing

I hope you have enjoyed this series on Back EMF. As I stated up-front, I really don't see anything in this article series that would directly benefit a loudspeaker designer. I had just never seen anybody examine this activity. The important lesson to be taken from this article series applies not just to Back EMF, but to virtually all areas of loudspeaker design. There are many areas of loudspeaker design that involve frequency-dependant (complex) arithmetic. Back EMF is a filter. Bass enclosures are filters. Crossover networks are filters. Moving-coil drivers are filters.

An understanding of, and ability to perform, calculations using complex numbers can empower a designer in many ways. I developed my skills using complex impedance information in an effort to design accurate passive crossovers using fewer components, for my own selfish ends. My efforts were successful, and my passive crossovers are very good.

I never would have noticed how Back EMF behaves had it not been for this work on passive crossovers. An understanding of complex impedances and reactive filters enabled me to figure out how Back EMF behaves, and increased my understanding in other areas, as well. Time permitting, I intend to share some of the techniques I've developed to build crossovers in future articles. aX

Editor's Note: All audioXpress *articles from 2001 to present can be found on the aX Cache, a USB drive available from www.cc-webshop.com.*

Why We Use "RMS" Current

Most people are aware that "RMS" stands for "root-mean-square." But that's not particularly illuminating. Here's why RMS quantities are so practical to use in the real world.

Ohm's Law is fundamentally about DC quantities: voltages, currents, and resistances (E, I, and R):

$I = E/R$
$E = IR$
$R = E/I$

Derivations of Ohm's Law pertaining to power or energy/time (E) are:

$E = I^2R$
$E = E^2/R$

When dealing with AC, we could express quantities, such as voltage, in terms of peak voltage, peak-to-peak voltage, average voltage, or RMS voltage. So why do we so often use RMS when discussing AC quantities?

The answer is that when we are using RMS quantities, which are equal to 0.707 of peak, the simple equations of Ohm's law can be used for AC, just as they are used for DC. If we were to represent the same activity using, for example, peak voltages, the Ohm's law equations could no longer be used to describe the relationships between voltage, current, resistance, and power, without using correcting factors. Using RMS quantities makes our lives easier, which is why we like to express these quantities using RMS. However, it should be noted that this only applies to the resistive component of reactive impedances.