# Back EMF Phase Relationships in Moving-Coil Loudspeakers (Part 1)

## Identifying, Quantifying, and Modeling Back EMF

As a loudspeaker's voice coil travels through the magnetic field in the gap, a voltage is produced, referred to as an electromotive force (EMF). Because it impedes input, the adjective "back" is used when referring to this opposing voltage in electric motors, including moving-coil loudspeakers. In a loudspeaker, the phase relationships between the input signal and the Back EMF cause the impedance of the unit to be inductive, resistive, or capacitive, depending on frequency. This article will explain the sinusoidal relationships that make this so.

By
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To begin, let me make it clear that this article is not about loudspeaker design. These observations might not be particularly useful to any loudspeaker designer, including myself. Its purpose is to point out something I noticed about how moving-coil drivers work, when visualizing the position of a speaker's moving-assembly lagging in time behind its input signal.

I had been considering how it was possible for the impedance of a driver to be purely resistive at resonance, such as is that of a DC rotary motor. In a moment of clarity, as I visualized the phase relationship at that frequency graphically, I realized that the resistive impedance itself revealed where the cone had to be at any instant in order for the impedance to be resistive.

After that realization at resonance, which is the simplest permutation, I had only to "slow things down," by doing "thought experiments," to see the similar relationships below resonance, and "speed things up" to reveal the driver's behavior above resonance.

After I qualitatively realized what was happening, it was then a matter of reconciling my observations and impedance measurements with what I know about reactive impedances and Newtonian physics. By the time I finished, it all seemed to make sense, and I present it for your perusal.

I hope this article will successfully communicate the only description of this activity I've seen in my 50-plus years of paying attention.

When originally completed, this article was five times as long. On the advice of people I trust, I present this condensed version. I hope you can get a fairly clear picture of what I'm attempting to convey. You might notice subtle references to content not otherwise in evidence.

If it seems as though anything has been under-explained or unsupported (including, but not limited to, the restorative force of the cone's suspension or voice-coil self-inductance), it has. If necessary, I can post further content online, as necessary, or could respond to questions, time permitting.

### Abstract

In this article, we explore the generation of Back EMF in the region of a loudspeaker's resonance. It examines EMF produced in a rotary DC generator, and expands to model the Back EMF in a DC motor.

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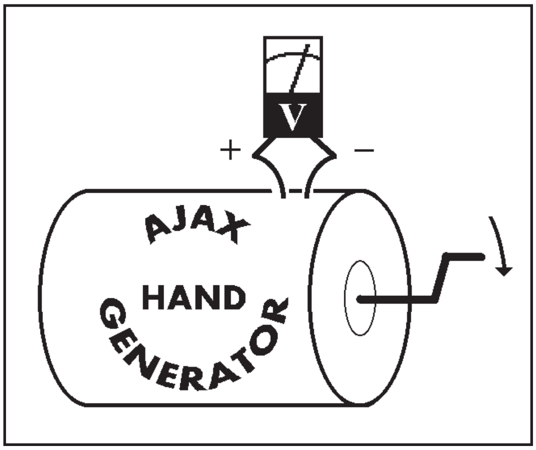



Figure 1: This is an example of a hand-crank powered DC generator and a voltmeter.

It then discusses the loudspeaker as a linear AC generator, before considering it as an AC-driven moving-coil loudspeaker. It then shows how to model the Back EMF of a driver with a parallel LCR filter, and specifies the values of the components for a particular woofer. In the article, I also demonstrate a method for calculating the position of the moving assembly, or "cone," relative to input signal, using complex impedance measurements.

The second part of this article defines the phase relationship between input current and cone position as "slip." It then explains how the degree of slip can be used to explain how and why the Back EMF of a driver manifests itself as an inductive reactance below resonance, a resistance at resonance, and a capacitive reactance above resonance.

The article also illustrates how and why the Back EMF at no time "drives" a current, but at most, shifts the phase of input current relative to input voltage, just as any reactive impedance.

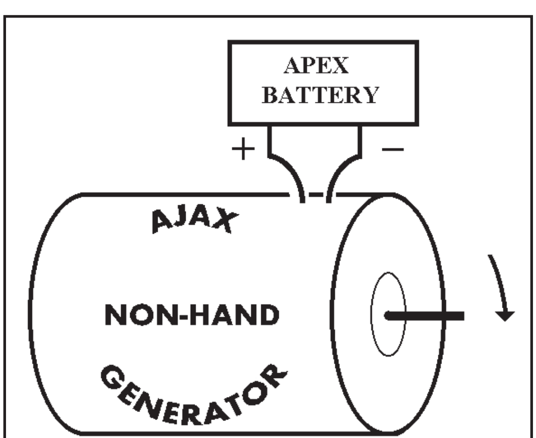



Figure 2: A "Generator" driven by an external voltage is a motor.

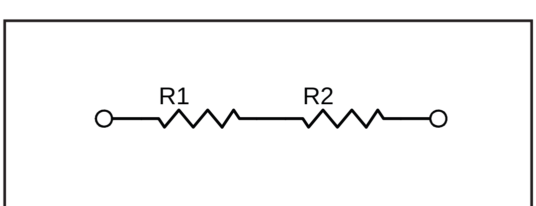



Figure 3: The DC motor is represented as two resistors.

## Precis

In a simple DC generator, wire coils turn on an axis, through a magnetic field, and produce a voltage (V = Blv). The armature travels in a constant direction, and the voltage thus produced is DC. This DC voltage drives a current though the resistance of the coils, in series with whatever external load is present.

When the same device is driven by an applied DC voltage, as opposed to an external physical force, the generation of EMF is unchanged (V = Blv), but the EMF produced opposes the input current, just as if it were a resistor. For this reason it is referred to as a "back" EMF, and can be modeled as a resistor.

Therefore, the DC motor can be modeled by using two resistors in series, one being the resistance of the coils, and the other being the resistance of the Back EMF.

Just as an external physical force can turn a rotary DC generator to produce a DC voltage, a loudspeaker driver can be driven by an external driving force to produce an AC voltage. When we do so, we can observe that the constantly changing position of the cone has a fixed 90° derivative relationship to its velocity (rate-of-change-of-position), which is phase with resulting EMF. Velocity precedes position.

Just as the Back EMF present in the DC motor presents a resistance to input voltage, and thus can be modeled by a resistor, the Back EMF in a moving-coil driver (loudspeaker) presents an impedance to input voltage, and can be modeled by an RCL parallel filter in series with the loudspeaker's DC resistance. Just as the Back EMF of the resistor in the DC motor has a specific value, these reactive components have specific, identifiable values.

Due to inertia, the position of the loudspeaker's cone lags behind input current. We will define this delay in time as "slip," which can be expressed as a phase angle. The precise phase angle between the two can be calculated from a driver's complex impedance information.

Degree of slip at a given frequency is what determines the magnitude and phase of the Back EMF, which opposes input voltage. This causes the phase angle of the impedance of the Back EMF to be inductive below resonance, resistive at resonance, and capacitive above resonance.

## EMF in a Simple DC Generator

A loudspeaker driver is a motor, albeit an AC linear motor (to and fro). We will start by comparing it to a less complex motor: The DC rotary motor (round and round). Even more fundamentally, we will begin by looking at the DC motor as a generator, when driven externally by a hand-crank.

In a rotary DC generator, the wire coils turn on an axis, through a magnetic field, and produce a voltage:

$$V=Blv \tag{1}$$

where:

V = voltage generated
B = magnetic field strength
l = length of conductor crossing magnetic field
v = velocity at which conductor moves through the magnetic field.

The armature travels at constant angular velocity, and the voltage is proportional to velocity. This DC voltage drives a current though the resistance of the coils, in series with any external load (see **Figure 1**).

## DC Motor

When the same device is driven by a DC voltage, as opposed to an external physical force, the generation of EMF is the same (V = Blv).

In the presence of a driving voltage, the EMF produced resists the input current, just as if it were a resistor (see **Figure 2**). For this reason, it is referred to as a "back" EMF, and can be modeled with a resistor. Consequently, the DC motor can be modeled by using two resistors in series, one being the resistance of the coils, and the other being the resistance of the Back EMF (see **Figure 3**).

The total resistance of the DC motor is equal to the internal resistance of the motor plus the effective resistance of the Back EMF generated as the motor turns. Back EMF manifests itself as, and can be expressed and measured as, a resistance. When this resistance is added to the motor's inherent resistance the sum equals the total resistance of the working motor.

$$R_1 + R_2 = R_t \tag{2}$$

where:

$R_1$ = motor internal resistance
$R_2$ = resistance of back EMF
$R_t$ = total resistance to DC voltage source

## First Phase Relationship: AC Linear Generator Creates Alternating EMF

Just as an external physical force (a hand-crank) can turn a rotary generator to produce a DC voltage, a loudspeaker driver can be driven by an external "wheel-and-stick" mechanism to produce an AC voltage. When we produce an AC voltage in this way, we can observe that the constantly changing position of the cone has a fixed 90° derivative relationship to its velocity (rate-of-change-of-position): velocity precedes position.

## Qualitative Description

We drove our DC generator with a hand-crank. For the purpose of comparison, we will drive an AC generator using an alternating external source.

**Figure 4** shows the AC generator being driven by a shaft connected to a wheel. One end of the shaft is fastened to a fixed point on the wheel, and the other end is attached to the moving assembly of the AC linear generator.

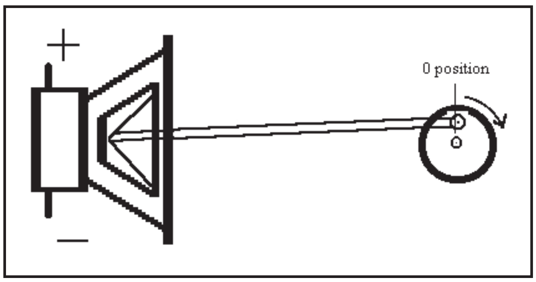



Figure 4: Here, the AC generator is driven by a wheel and a shaft (not drawn to scale).

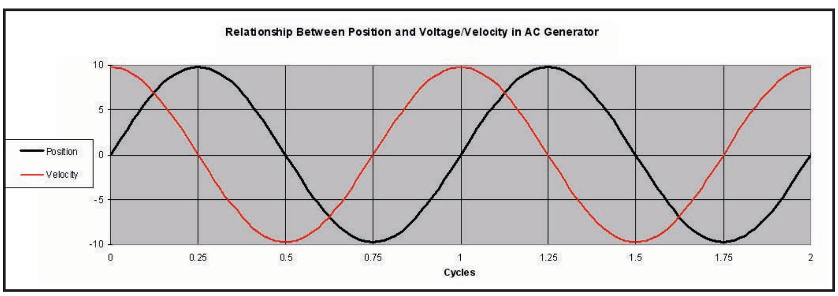



Figure 5: The graph shows the phase relationship between the cone position, the velocity, and the voltage.

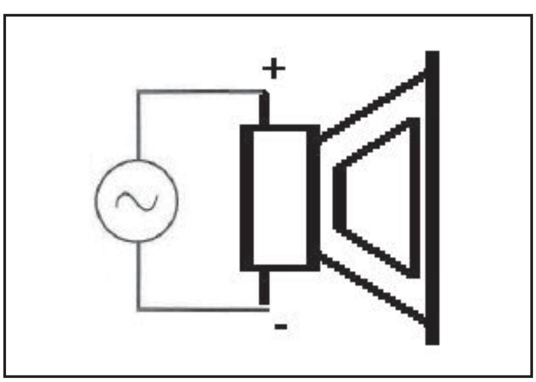



Figure 6: In the moving-coil loudspeaker, the AC linear generator is driven by external voltage.

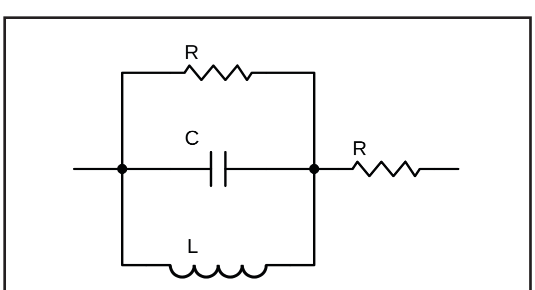



Figure 7: The LCR filter represents Back EMF and the resistor represents the voice coil DC resistance.

## Phase Relationship in Linear AC Generator

Back EMF can be understood using differential and integral sinusoidal relationships. This will illustrate the first of them.

Note that the wheel driving the shaft (shown in Figure 4) has a designated zero position. This zero position corresponds to the moving assembly's point of zero excursion. Importantly, when the wheel is turning, this is also the moving assembly's point of maximum or minimum (negative) velocity, as the moving assembly travels back and forth. In other words, there is a 90° phase-shift between position and velocity. Velocity is in phase with the instantaneous voltage produced by the generator.

The position of the cone at any instant is a given, based on the position of the wheel driving the shaft. The first derivative of position is velocity, which is defined to be rate-of-change of position, and is represented by a cosine curve, as shown in **Figure 5**, and voltage generated at any instant is proportional to the cone's velocity. You can see that the point of zero position coincides with the point of maximum (absolute) velocity. We will see that the moving assembly of the voltage-driven loudspeaker driver has the same relationship of position to velocity.

## AC Linear Motor: The Moving-Coil Loudspeaker

Just as our DC generator can become a motor when an external DC voltage is applied, our AC generator becomes an AC motor when exposed to an AC voltage source, such as an audio amplifier. We can use such a device to reproduce sound. We commonly refer to an AC motor used in this fashion as a moving-coil loudspeaker (see **Figure 6**).

And, as the Back EMF produced by a rotary DC motor can be modeled by using a resistor, the Back EMF produced by the AC loudspeaker driver can be modeled by using an LCR filter. The single resistor representing the voice coil's resistance is analogous to the resistance of the coils in the DC motor, and the Back EMF is represented by an inductor, a capacitor, and a resistor in parallel with one another, as shown in **Figure 7**.

In the region of its resonance, the impedance of the loudspeaker is the vector sum of the impedance of this LCR filter and the DC resistance of the voice coil. At any frequency, impedance can be described by its magnitude and phase angle. We will use a specific woofer when describing the behavior of Back EMF, the impedance characteristics of which are illustrated in **Figure 8**.

We use a specific real-world woofer to illustrate the concepts discussed in this paper. It is an 8" woofer from a Fisher system built in about 1965. It is a good quality driver. The characteristics of this woofer that are relevant to the article are shown in **Table 1**.

## Calculate Magnitude and Phase of Back EMF from Impedance Measurements

We will calculate the impedance and phase angle of the Back EMF at three different frequencies: below resonance ($f_1$), at the resonant frequency ($f_s$), above resonance ($f_2$). The frequencies below and above resonance are the frequencies $f_1$ and $f_2$, which are commonly used when measuring a driver to calculate its motor parameters, $Q_{TS}$, $Q_{ES}$, and $Q_{MS}$.[1] The choice of these frequencies, while entirely arbitrary, is interesting because of the symmetrical nature of the impedances of at $f_1$ and $f_2$, as will be seen.

Because these impedances are complex, having a magnitude and a phase angle, we will use complex numbers to calculate them, and vector diagrams to illustrate them.

## Determination of Magnitude and Phase Angle of Back EMF Below Resonance: $f_1$

The frequency $f_1$ for this driver is 23 Hz. We will illustrate how to isolate Back EMF at this frequency (see **Figure 9**). As was the case with the DC motor, we will subtract the DC resistance of the wire windings from the total impedance, in order to solve for the impedance of the Back EMF:

Frequency: 23 Hz

The magnitude and phase angle of the driver's impedance at this frequency were measured using test equipment:

Impedance Z: 13.64 Ω
Impedance Phase Angle (RAD): 1.07 or 61.3°

This complex impedance has both an inductive component ($X_l$), and a resistive c omponent ($R_t$).

$X_l$ = 13.64 sin(1.07) = 11.97 Ω
$R_t$ = 13.64 cos(1.07) = 6.55 Ω

Therefore, the complex impedance, can be expressed and modeled using these values:

Complex Impedance (ohms): 6.55 + 11.97j

Next, as we did with the DC motor, we subtract

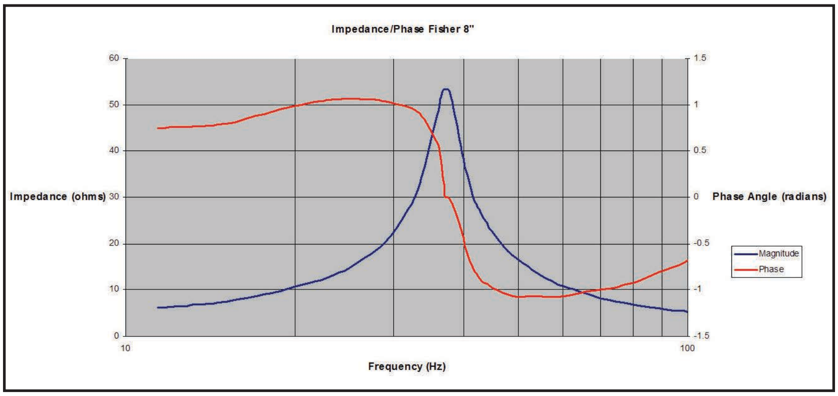



Figure 8: The impedance curve of the Fisher woofer shows the magnitude and the phase.

| Fisher 8" Woofer Characteristics | |
|---|---|
| $F_s$ | 36.93 Hz |
| DCR | 3.50 Ω |
| $Z_{MAX}$ | 53.50 Ω |
| $f_1$[1] | 23 Hz |
| Impedance at $f_1$ | 13.64 Ω, at 61.3° phase angle |
| $f_2$[1] | 59.30 Hz |
| Impedance at $f_2$ | 13.64 Ω, at -61.3° phase angle |

Table 1: These are characteristics of the Fisher 8" woofer that are relevant for this article.

the DC resistance of the voice coil to solve for Back EMF, this time using complex numbers:

Voice Coil DC Resistance $R_{DC}$: 3.5 Ω

The DC resistance is, by definition, non-reactive, and therefore, has a phase angle of zero.

Complex Expression of $R_{DC}$ (ohms): 3.5 + 0j

So, magnitude and phase of Back EMF can be found by subtraction:

Back EMF = Z - $R_{DC}$ = (6.55 + 11.97j) – (3.5 + 0j) = (3.05 + 11.97j)

Complex Expression of Back EMF $Z_{EMF}$ (ohms): 3.05 + 11.97j

The Back EMF at this frequency has a resistance ($R_{EMF}$) of 3.05 Ω, and an inductive reactance ($X_l$) of 11.97 Ω. In order to determine the net value of the Back EMF, we use the Pythagorean Theorem:

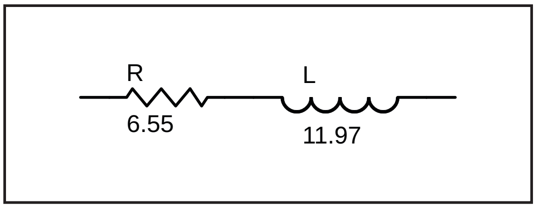
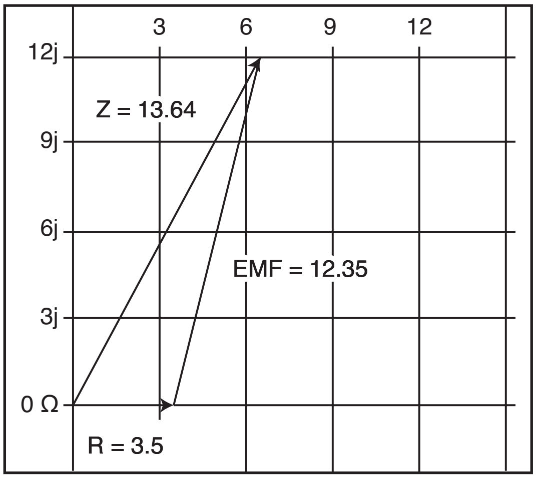



Figure 9: The impedances of the resistor and the inductor model are shown at 23 Hz.




Figue 10: The vector diagram summarizes impedance at $f_1$, below resonance.

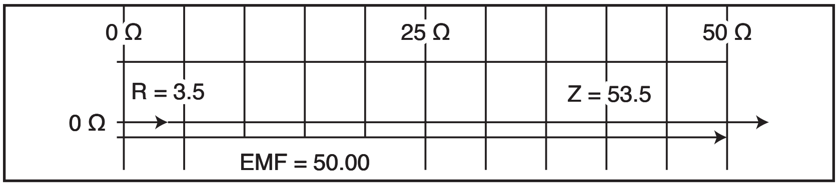



Figure 11: The vector diagram summarizes resistance impedance at Resonance ($f_s$): $R_{DC}$ + EMF = Z

$$Z_{EMF} = \sqrt{\left(R_{EMF}{}^2\right) + \left(X_l{}^2\right)}$$

Or, in this case,

$$\sqrt{\left(3.05^2\right) + \left(11.97^2\right)} = 12.35$$

Net Impedance of Back EMF: 12.35 Ω

The phase angle has a trigonometric relationship:

Phase Angle = atan($X_l$/$R_{EMF}$)

Or, in this case,

Phase Angle = atan(11.97/3.05)
Back EMF Phase Angle (RAD): 1.32 or 75.6°

**Figure 10** shows the vector diagram of the unit's impedance at $f_1$, with R and EMF adding up to Z.

## Determination of Magnitude and Phase Angle of Back EMF at Frequency of Resonance: $f_s$

The frequency of resonance ($f_s$) of our Fisher woofer is 36.93 Hz. At resonance, just as at $f_1$, we can subtract the DC resistance of the voice-coil windings from the total impedance in order to solve for Back EMF.

As before, the impedance of the driver at this frequency was measured on the bench:

Impedance (Z): 53.50 Ω
Impedance Phase Angle: 0

Unlike the reactive impedance at $f_1$, this impedance is purely resistive, and therefore, has a phase angle of zero. As with the rotary motor, we can describe the voice coil resistance as $R_1$, the Back EMF as $R_s$, and the total impedance as ($R_t$):

$$R_1 + R_2 = R_t \qquad (2)$$

Because the voice-coil resistance and the impedance of the Back EMF have the same phase angle (zero), the magnitude of the Back EMF can be found with a simple arithmetic subtraction, just as with the rotary motor:

$$R_{EMF} = R_t - R_1 \qquad (3)$$

or,

$$R_{EMF} = 53.5 - 3.5 = 50\ \Omega$$

The impedance of the Back EMF at 36.93 Hz is 50 Ω, purely resistive. **Figure 11** shows the vector diagram of the unit's impedance at resonance ($f_s$). Again, R and EMF add up to Z. At resonance, the impedance is purely resistive. This tells us that Back EMF is resistive. More on this later.

## Determination of Magnitude and Phase Angle of Back EMF above Resonance: $f_2$

These calculations are a mirror-image of those performed at $f_1$, below resonance. The form of the calculations is the same. For that reason, and for the sake of brevity, we will only provide the results and the vector diagram.

The frequency $f_2$ for this driver is 59.30 Hz. We will subtract the DC resistance of the voice coil windings from the total impedance, at this frequency, as we have done at the lower frequencies.

Back EMF Phase Angle (RAD): -1.32 or -75.6°

**Figure 12** shows the vector diagram of the unit's impedance at $f_2$, with R and EMF adding up to Z. Above resonance, the impedance is decreasing

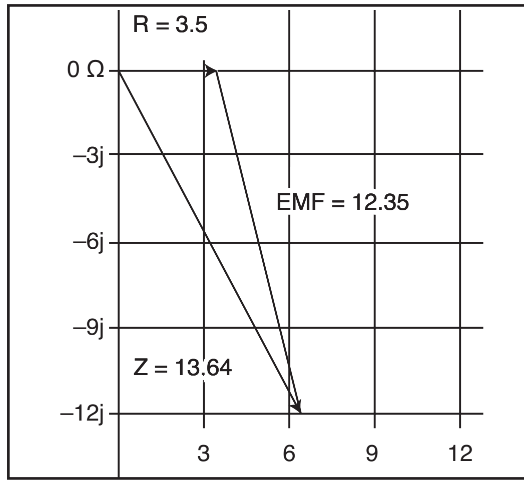



Figure 12: The vector diagram summarizes impedance at $f_2$, above resonance.

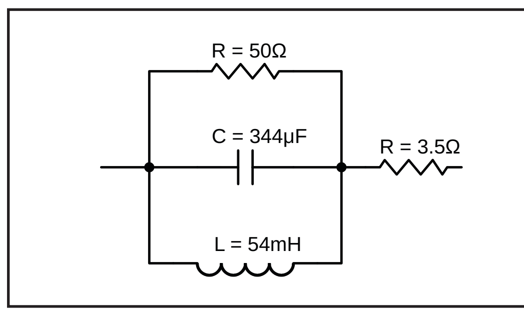



Figure 13: Here are equivalent real-world values of impedance for the Fisher woofer.

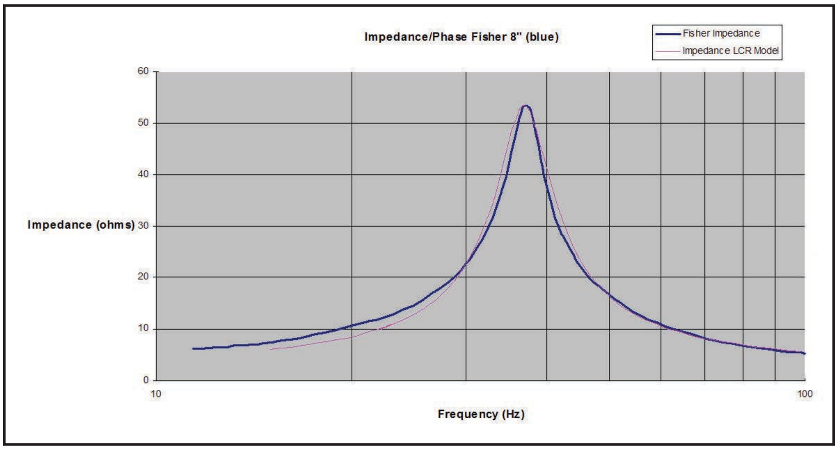



Figure 14: The graph shows the impedance curves of the Fisher woofer and the equivalent LCR filter.

with respect to frequency. This tells us that the impedance and the Back EMF are capacitive.

## LCR Model of Fisher Woofer

In the section on the DC motor, we were able to model the resistance and Back EMF using resistors in series. We were able to quantify the Back EMF by subtracting the coil resistance from the total resistance. Similarly, the Back EMF of our Fisher woofer can be modeled using specific components, as follows:

Effective Resistance of Back EMF: 50 Ω

Effective Capacitance of Back EMF: 344 µF

Effective Inductance of Back EMF: 54 mH

**Figure 13** shows the LCR filter model, with the effective component values. This filter's impedance curve is comparable to that of the Fisher woofer. **Figure 14** shows the impedances both of the modeled filter and the actual woofer.

## Intersection of Mass/Compliance and Back EMF as Modeled

As it happens,

$$LC = MC_{MS} \qquad (4)$$

where,

L = inductor in LCR model
C = capacitor in LCR model
M = mass of loudspeaker moving assembly
$C_{MS}$ = loudspeaker's compliance

## Back EMF Voltage and Stall Ratio

Because Back EMF manifests itself as impedance, we have been expressing its magnitude in ohms. Ultimately, however, Back EMF is a voltage, and it can be interesting to consider it as such. The ratio of the Back EMF to input voltage is equal to the ratio of Back EMF impedance to total impedance, just as with any impeding series components. This can give us an idea of the significance of Back EMF at a given frequency. First we will calculate this ratio at $F_1$, 23 Hz, for an input voltage of 10 V.

- Net Loudspeaker Impedance: 13.64 Ω (see "Linear Motor")
- Current at 10 V: 7.33 A (I = E/Z)
- Back EMF Impedance: 12.35 Ω (see "Linear Motor")
- Back EMF Voltage: 9.05 V
- Ratio Back EMF to Input Voltage: 0.905

So, at this frequency, for a 10 V input, we produce a Back EMF of 9.05 V.

At resonance, 39.93 Hz, the current is diminished by a high resistive Back EMF, but the magnitude of the Back EMF is barely changed:

Impedance: 53.5 Ω
Current: 0.187 A (I = E/Z)
Back EMF Impedance: 50 Ω
Back EMF: 9.35 V
Ratio Back EMF to Input Voltage: 0.935

Interestingly, we see a change in Back EMF of roughly 3%. (9.05 V to 9.35 V). For a given input signal, the magnitude of the Back EMF is nearly unchanged with frequency, with this woofer. (These figures will be duplicated at $f_2$, above resonance, where the numbers are the same, but the phase angles are mirror-image.)

## Stall Ratio

Bob Carver and others have found it useful to express Back EMF in volts, and have defined a "stall ratio," which is seen as related to efficiency of energy conversion.[2]

$$\textit{Stall Ratio} = \frac{V_{EMF}}{V_I - V_{EMF}} \qquad (5)$$

where,

$V_{EMF}$ = voltage of Back EMF, and
$V_I$ = applied input voltage

"Stall," would be a condition occurring when the driver operates with a stall ratio of less than 1, and the Back EMF is less than half the applied voltage. The stall ratio of this woofer at $f_1$ and $f_2$ is 9.53, and at $f_s$ is 8.7. Both figures are well above Carver's benchmark figure of 1.

## Next Month

The second part of this article series will define the delay in time between input current and moving-assembly position as "Slip," and examine how, because Slip increases with frequency, the resulting Back EMF manifests itself as it does in its three different modes of generation: below resonance, at resonance, and above resonance.

## About the Author

Andy Lewis lives in Englewood, CO, plays drums, and produces "Acme Sound Low-B" low-coloration loudspeakers for bass players. He is also involved in Special Olympics and other disability activities with his son, Collin.

## References

[1] R. Small, "Director Radiator Loudspeaker System Analysis," *Journal of the Audio Engineering Society* (*JAES*), June 1972.

[2] I. Berger, "The Stall Ratio and Back EMF," embedded in "Sunfire True Subwoofer Mark II, D.B. Keele, *Audio*, November 1997.